(20) 1. Let $v_1, v_2, \ldots, v_k$ be linearly independent vectors in $\mathbb{F}^n$.

(a) Prove that $k \leq n$.

(b) Prove that if $k = n$, then $\{v_1, v_2, \ldots, v_k\}$ is a basis of $\mathbb{F}^n$.

(20) 2. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$ and take $\mathbb{F} = \mathbb{R}$.

(a) Find the eigenvalues of A and find an ordered basis of $\mathbb{R}^2$ consisting of eigenvectors of A.

(b) Write down an invertible matrix Q and a diagonal matrix D such that $Q^{-1}AQ = D$. (You do *not* have to calculate Q^{-1}.)

(20) 3. Let T be a linear operator on a finite-dimensional vector space V over $\mathbb{F}$.

(a) Prove that T is invertible if and only if 0 is not an eigenvalue of T.

(b) Prove that if T is invertible, then a scalar $\lambda \in \mathbb{F}$ is an eigenvalue of T if and only if λ^{-1} is an eigenvalue of T^{-1}.

(20) 4. Let $V = P_2(\mathbb{R})$, define the linear operator $T : V \to V$ by $T(f(x)) = f'(x) - f(1)$ for $f(x) \in V$, and let W be the T-cyclic subspace of V generated by the polynomial $x^2 - 1 \in V$.

(a) Find an ordered basis for W.

(b) Find the characteristic polynomial of the restriction T_W of T to W.

(20) 5. NOTE: PARTS (B) AND (C) OF THIS PROBLEM ARE ON THE NEXT PAGE.

Let $A = \begin{pmatrix} 0 & -4 \\ 1 & 4 \end{pmatrix}$ in $M_{2\times 2}(\mathbb{R})$.

(a) Find an eigenvector for A.

CONTINUATION FROM THE PREVIOUS PAGE

(b) *Use your answer to (a)* to find a Jordan canonical basis for the linear operator L_A. In other words, find an ordered basis of $\mathbb{R}^2$ which is a cycle of generalized eigenvectors of L_A.

(c) Write down the Jordan canonical form J of A and an invertible matrix Q such that $Q^{-1}AQ = J$. (You do *not* have to calculate Q^{-1}.)

MATH 350:02, EXAM 2

April 13, 2022

NAME (*please print*): ______________________________

SIGNATURE: ______________________________

Do all 5 problems.

Show all your work and justify your answers.

Problem number	Possible points	Points earned (out of 100):
1	20	
2	20	
3	20	
4	20	
5	20	
Total points earned:		