MATH 350 Linear Algebra
Homework 7

Instructor: Will Chen

November 1, 2022

Problems

Book Problems 2 points each, 26 points total

- Section 5.1, Problems 4(d), 5(b), 5(f), 9(a), 9(b), 10
- Section 5.2, Problems 2(b), 2(d), 3(d), 9(b), 11(a), 11(b), 13

  For 11(a), recall that $\mathrm{tr}(A)$ denotes the trace of the matrix A, which is defined to be the sum of the diagonal entries.

Additional Problems (2 points each, 4 points total)

- For each of the following matrices $A \in M_2(F)$, determine all eigenvalues of A. Then, for each eigenvalue λ of A, find the set of eigenvectors corresponding to λ. Then, if possible, find a basis for F^n consisting of eigenvectors of A. If successful in finding such a basis, determine an invertible matrix Q and a diagonal matrix D such that $Q^{-1}AQ = D$.

  Do the above for the "rotation by 60° matrix" $A = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ with $F = \mathbb{R}$. Then do the above for the same matrix but with $F = \mathbb{C}$.

- Suppose $A, B \in M_n(F)$ are similar. Recall that this means that there is an invertible matrix Q such that $B = QAQ^{-1}$. Prove that $\chi_A(t) = \chi_B(t)$.